

CPT and effective Hamiltonians for neutral kaon and similar complexes*

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Abstract

We begin with a discussion of the general form and general CP- and CPT- transformation properties of the Lee-Oehme-Yang (LOY) effective Hamiltonian for the neutral kaon complex. Next, the properties of the exact effective Hamiltonian for this complex are discussed. Using the Khalfin Theorem we show that the diagonal matrix elements of the effective Hamiltonian governing the time evolution in the subspace of states of an unstable particle and its antiparticle need not be equal at for $t > t_0$ (t_0 is the instant of creation of the pair) when the total system under consideration is CPT invariant but CP noninvariant. The unusual consequence of this result is that, contrary to the properties of stable particles, the masses of the unstable particle "1" and its antiparticle "2" need not be equal for $t \gg t_0$ in the case of preserved CPT and violated CP symmetries. We also show that there exists an approximation which is more accurate than the LOY, and which leads to an effective Hamiltonian whose diagonal matrix elements possess properties consistent with the conclusions for the exact effective Hamiltonian described above.

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1 Introduction.

The problem of testing CPT-invariance experimentally has attracted the attention of physicist, practically since the discovery of antiparticles. CPT symmetry is a fundamental theorem of axiomatic quantum field theory which follows from locality, Lorentz invariance, and unitarity [1]. Many tests of CPT-invariance consist in searching for decay process of neutral kaons. All known CP- and hypothetically possible CPT-violation effects in the neutral kaon complex are described by solving the Schrödinger-like evolution equation [2] — [9] (we use $\hbar = c = 1$ units)

$$i\frac{\partial}{\partial t}|\psi; t>_{\parallel} = H_{\parallel}|\psi; t>_{\parallel} \quad (1)$$

for $|\psi; t>_{\parallel}$ belonging to the subspace $\mathcal{H}_{\parallel} \subset \mathcal{H}$ (where \mathcal{H} is the state space of the physical system under investigation), e.g., spanned by orthonormal neutral kaons states $|K_0>$, $|\overline{K}_0>$, and so on, (then states corresponding to the decay products belong to $\mathcal{H} \ominus \mathcal{H}_{\parallel} \stackrel{\text{def}}{=} \mathcal{H}_{\perp}$), and nonhermitian effective Hamiltonian H_{\parallel} obtained usually by means of the Lee-Oehme-Yang (LOY) approach (within the Weisskopf-Wigner approximation (WW)) [2] — [5], [9]:

$$H_{\parallel} \equiv M - \frac{i}{2}\Gamma, \quad (2)$$

where

$$M = M^+, \quad \Gamma = \Gamma^+, \quad (3)$$

are (2×2) matrices.

The solutions of Eq. (1) can be written in matrix form and such a matrix defines the evolution operator (which is usually nonunitary) $U_{\parallel}(t)$ acting in \mathcal{H}_{\parallel} :

$$|\psi; t>_{\parallel} = U_{\parallel}(t)|\psi; t_0 = 0>_{\parallel} \stackrel{\text{def}}{=} U_{\parallel}(t)|\psi>_{\parallel}, \quad (4)$$

where,

$$|\psi>_{\parallel} \equiv q_1|\mathbf{1}> + q_2|\mathbf{2}>, \quad (5)$$

and $|\mathbf{1}>$ stands for the vectors of the $|K_0>$, $|B_0>$ type and $|\mathbf{2}>$ denotes antiparticles of particle "1": $|\overline{K}_0>$, $|\overline{B}_0>$, $\langle \mathbf{j}|\mathbf{k}> = \delta_{jk}$, $j, k = 1, 2$.

In many papers it is assumed that the real parts, $\Re(\cdot)$, of the diagonal matrix elements of H_{\parallel} :

$$\Re(h_{jj}) \equiv M_{jj}, \quad (j = 1, 2), \quad (6)$$

where

$$h_{jk} = \langle \mathbf{j} | H_{\parallel} | \mathbf{k} \rangle, \quad (j, k = 1, 2), \quad (7)$$

correspond to the masses of particle "1" and its antiparticle "2" respectively [2] — [9], (and such an interpretation of $\Re(h_{11})$ and $\Re(h_{22})$ will be used in this paper), whereas the imaginary parts, $\Im(\cdot)$,

$$-2\Im(h_{jj}) \equiv \Gamma_{jj}, \quad (j = 1, 2), \quad (8)$$

are interpreted as the decay widths of these particles [2] — [9]. Such an interpretation seems to be consistent with the recent and the early experimental data for the neutral kaon and similar complexes [10].

Relations between matrix elements of H_{\parallel} implied by CPT–transformation properties of the Hamiltonian H of the total system, containing neutral kaon complex as a subsystem, are crucial to designing CPT–invariance and CP–violation tests and to proper interpretation of their results. The aim of this paper is to examine the properties of the exact H_{\parallel} generated by the CPT–symmetry of the total system under consideration and independent of the approximation used and to compare these properties of the exact and of the approximate H_{\parallel} .

2 H_{LOY} and CPT–symmetry.

Now, let us consider briefly some properties of the LOY model. Let H be the total (selfadjoint) Hamiltonian, acting in \mathcal{H} — then the total unitary evolution operator $U(t)$ fulfills the Schrödinger equation

$$i \frac{\partial}{\partial t} U(t) |\phi \rangle = H U(t) |\phi \rangle, \quad U(0) = I, \quad (9)$$

where I is the unit operator in \mathcal{H} , $|\phi \rangle \equiv |\phi; t_0 = 0 \rangle \in \mathcal{H}$ is the initial state of the system:

$$|\phi \rangle \equiv |\psi \rangle_{\parallel} \quad (10)$$

in our case $|\phi; t \rangle = U(t) |\phi \rangle$. Let P denote the projection operator onto the subspace \mathcal{H}_{\parallel} :

$$P\mathcal{H} = \mathcal{H}_{\parallel}, \quad P = P^2 = P^+, \quad (11)$$

then the subspace of decay products \mathcal{H}_{\perp} equals

$$\mathcal{H}_{\perp} = (I - P)\mathcal{H} \stackrel{\text{def}}{=} Q\mathcal{H}, \quad Q \equiv I - P. \quad (12)$$

For the case of neutral kaons or neutral B -mesons, etc., the projector P can be chosen as follows:

$$P \equiv |\mathbf{1}\rangle\langle\mathbf{1}| + |\mathbf{2}\rangle\langle\mathbf{2}|, \quad (13)$$

and the definition of $|K_0\rangle$ and $|\overline{K}_0\rangle$ is analogous to the one used in the LOY theory for corresponding vectors. In the LOY approach it is assumed that vectors $|\mathbf{1}\rangle$, $|\mathbf{2}\rangle$ considered above are eigenstates of $H^{(0)}$ for a 2-fold degenerate eigenvalue m_0 :

$$H^{(0)}|\mathbf{j}\rangle = m_0|\mathbf{j}\rangle, \quad j = 1, 2, \quad (14)$$

where $H^{(0)}$ is the so called free Hamiltonian, $H^{(0)} \equiv H_{strong} = H - H_W$, and H_W denotes weak and other interactions which are responsible for transitions between the eigenvectors of $H^{(0)}$, i.e., for the decay process. This means that

$$[P, H^{(0)}] = 0. \quad (15)$$

The condition guaranteeing the occurrence of transitions between subspaces \mathcal{H}_{\parallel} and \mathcal{H}_{\perp} , i.e., the decay process of states in \mathcal{H}_{\parallel} , can be written as follows

$$[P, H_W] \neq 0, \quad (16)$$

that is

$$[P, H] \neq 0. \quad (17)$$

Usually, in LOY and related approaches, it is assumed that

$$\Theta H^{(0)} \Theta^{-1} = H^{(0)+} \equiv H^{(0)}, \quad (18)$$

where Θ is the antiunitary operator:

$$\Theta \stackrel{\text{def}}{=} \mathcal{CPT}. \quad (19)$$

The subspace of neutral kaons \mathcal{H}_{\parallel} is assumed to be invariant under Θ :

$$\Theta P \Theta^{-1} = P^+ \equiv P. \quad (20)$$

In the kaon rest frame, the time evolution is governed by the Schrödinger equation (9), where the initial state of the system has the form (10), (5). Within assumptions (14) — (16) the Weisskopf–Wigner approach, which is

the source of the LOY method, leads to the following formula for H_{LOY} (e.g., see [2, 3, 4, 9]):

$$H_{LOY} = m_0 P - \Sigma(m_0) \equiv PHP - \Sigma(m_0), \quad (21)$$

$$= M_{LOY} - \frac{i}{2} \Gamma_{LOY} \quad (22)$$

where it has been assumed that $\langle \mathbf{1} | H_W | \mathbf{2} \rangle = \langle \mathbf{1} | H_W | \mathbf{2} \rangle^* = 0$ (see [2] — [9]),

$$\Sigma(\epsilon) = PHQ \frac{1}{QH Q - \epsilon - i0} QHP. \quad (23)$$

The matrix elements h_{jk}^{LOY} of H_{LOY} are

$$h_{jk}^{LOY} = H_{jk} - \Sigma_{jk}(m_0), \quad (j, k = 1, 2), \quad (24)$$

$$= M_{jk}^{LOY} - \frac{i}{2} \Gamma_{jk}^{LOY} \quad (25)$$

where, in this case,

$$H_{jk} = \langle \mathbf{j} | H | \mathbf{k} \rangle \equiv \langle \mathbf{j} | (H^{(0)} + H_W) | \mathbf{k} \rangle \equiv m_0 \delta_{jk} + \langle \mathbf{j} | H_W | \mathbf{k} \rangle, \quad (26)$$

and $\Sigma_{jk}(\epsilon) = \langle \mathbf{j} | \Sigma(\epsilon) | \mathbf{k} \rangle$.

Now, if $\Theta H_W \Theta^{-1} = H_W^+ \equiv H_W$, that is if

$$[\Theta, H] = 0, \quad (27)$$

then using, e.g., the following phase convention [3] — [9]

$$\Theta | \mathbf{1} \rangle \stackrel{\text{def}}{=} - | \mathbf{2} \rangle, \quad \Theta | \mathbf{2} \rangle \stackrel{\text{def}}{=} - | \mathbf{1} \rangle, \quad (28)$$

and taking into account that $\langle \psi | \varphi \rangle = \langle \Theta \varphi | \Theta \psi \rangle$, one easily finds from (21) – (26) that

$$h_{11}^{LOY\Theta} - h_{22}^{LOY\Theta} = 0, \quad (29)$$

and thus

$$M_{11}^{LOY} = M_{22}^{LOY}, \quad (30)$$

(where $h_{jk}^{LOY\Theta}$ denotes the matrix elements of H_{LOY}^Θ — of the LOY effective Hamiltonian when the relation (27) holds), in the CPT-invariant system. This is the standard result of the LOY approach and this is the picture which one meets in the literature [2] — [8].

If it is assumed that the CPT-symmetry is not conserved in the physical system under consideration, i.e., that

$$[\Theta, H] \neq 0, \quad (31)$$

then $h_{11}^{LOY} \neq h_{22}^{LOY}$.

It is convenient to express the difference between H_{LOY}^Θ and the effective Hamiltonian H_{LOY} appearing within the LOY approach in the case of nonconserved CPT-symmetry as follows

$$\begin{aligned} H_{LOY} &\equiv H_{LOY}^\Theta + \delta H_{LOY} \\ &= \begin{pmatrix} (M_0 + \frac{1}{2}\delta M) - \frac{i}{2}(\Gamma_0 + \frac{1}{2}\delta\Gamma), & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^*, & (M_0 - \frac{1}{2}\delta M) - \frac{i}{2}(\Gamma_0 - \frac{1}{2}\delta\Gamma) \end{pmatrix}. \end{aligned} \quad (32)$$

In other words

$$h_{jk}^{LOY} = h_{jk}^{LOY\Theta} + \Delta h_{jk}^{LOY}, \quad (33)$$

where

$$\Delta h_{jk}^{LOY} = (-1)^{j+1} \frac{1}{2} (\delta M - \frac{i}{2} \delta \Gamma) \delta_{jk}, \quad (34)$$

and $j, k = 1, 2$. Within this approach the δM and $\delta \Gamma$ terms violate the CPT-symmetry.

3 CPT and the exact effective Hamiltonian

The aim of this Section is to show that, contrary to the LOY conclusion (29), the diagonal matrix elements of the exact effective Hamiltonian $H_{||}$ can not be equal when the total system under consideration is CPT invariant but CP noninvariant. This will be done by means of the method used in [11].

The universal properties of the (unstable) particle-antiparticle subsystem of the system described by the Hamiltonian H , for which the relation (27) holds, can be extracted from the matrix elements of the exact $U_{||}(t)$ appearing in (4). Such $U_{||}(t)$ has the following form

$$U_{||}(t) = P U(t) P, \quad (35)$$

where P is defined by the relation (13), and $U(t)$ is the total unitary evolution operator $U(t)$, which solves the Schrödinger equation (9). Of course, $U_{||}(t)$ has a nontrivial form only if (17) holds, and only then transitions of states

from \mathcal{H}_{\parallel} into \mathcal{H}_{\perp} and vice versa, i.e., decay and regeneration processes, are allowed.

Using the matrix representation one finds

$$U_{\parallel}(t) \equiv \begin{pmatrix} \mathbf{A}(t) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \quad (36)$$

where $\mathbf{0}$ denotes the suitable zero submatrices and a submatrix $\mathbf{A}(t)$ is the 2×2 matrix acting in \mathcal{H}_{\parallel}

$$\mathbf{A}(t) = \begin{pmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{pmatrix} \quad (37)$$

and $A_{jk}(t) = \langle \mathbf{j} | U_{\parallel}(t) | \mathbf{k} \rangle \equiv \langle \mathbf{j} | U(t) | \mathbf{k} \rangle$, ($j, k = 1, 2$).

Now, assuming (27) and using the phase convention (28), [2] — [5], one easily finds that [7], [12, 13, 15]

$$A_{11}(t) = A_{22}(t). \quad (38)$$

Note that assumptions (27) and (28) give no relations between $A_{12}(t)$ and $A_{21}(t)$.

The important relation between amplitudes $A_{12}(t)$ and $A_{21}(t)$ follows from the famous Khalfin's Theorem [7], [13] — [15]. This Theorem states that in the case of unstable states, if amplitudes $A_{12}(t)$ and $A_{21}(t)$ have the same time dependence

$$r(t) \stackrel{\text{def}}{=} \frac{A_{12}(t)}{A_{21}(t)} = \text{const} \equiv r, \quad (39)$$

there must be $|r| = 1$.

For unstable particles relation (38) means that the decay laws

$$p_j(t) \stackrel{\text{def}}{=} |A_{jj}(t)|^2, \quad (40)$$

(where $j = 1, 2$), of the particle $|\mathbf{1}\rangle$ and its antiparticle $|\mathbf{2}\rangle$ are equal,

$$p_1(t) \equiv p_2(t). \quad (41)$$

The consequence of this last property is that the decay rates of the particle $|\mathbf{1}\rangle$ and its antiparticle $|\mathbf{2}\rangle$ must be equal too.

From (38) it does not follow that the masses of particle "1" and the antiparticle "2" should be equal.

More conclusions about the properties of the matrix elements of $H_{||}$ one can infer analyzing the following identity [16] — [21]

$$H_{||} \equiv H_{||}(t) = i \frac{\partial U_{||}(t)}{\partial t} [U_{||}(t)]^{-1}, \quad (42)$$

where $[U_{||}(t)]^{-1}$ is defined as follows

$$U_{||}(t) [U_{||}(t)]^{-1} = [U_{||}(t)]^{-1} U_{||}(t) = P. \quad (43)$$

(Note that the identity (42) holds, independent of whether $[P, H] \neq 0$ or $[P, H] = 0$). The expression (42) can be rewritten using the matrix $\mathbf{A}(t)$

$$H_{||}(t) \equiv i \frac{\partial \mathbf{A}(t)}{\partial t} [\mathbf{A}(t)]^{-1}. \quad (44)$$

Relations (42), (44) must be fulfilled by the exact as well as by every approximate effective Hamiltonian governing the time evolution in every two dimensional subspace $\mathcal{H}_{||}$ of states \mathcal{H} [16] — [21].

It is easy to find from (44) the general formulae for the diagonal matrix elements, h_{jj} , of $H_{||}(t)$, in which we are interested. We have

$$h_{11}(t) = \frac{i}{\det \mathbf{A}(t)} \left(\frac{\partial A_{11}(t)}{\partial t} A_{22}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \right), \quad (45)$$

$$h_{22}(t) = \frac{i}{\det \mathbf{A}(t)} \left(-\frac{\partial A_{21}(t)}{\partial t} A_{12}(t) + \frac{\partial A_{22}(t)}{\partial t} A_{11}(t) \right). \quad (46)$$

Now, assuming (27) and using the consequence (38) of this assumption, one finds

$$h_{11}(t) - h_{22}(t) = \frac{i}{\det \mathbf{A}(t)} \left(\frac{\partial A_{21}(t)}{\partial t} A_{12}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \right). \quad (47)$$

Next, after some algebra one obtains

$$h_{11}(t) - h_{22}(t) = -i \frac{A_{12}(t) A_{21}(t)}{\det \mathbf{A}(t)} \frac{\partial}{\partial t} \ln \left(\frac{A_{12}(t)}{A_{21}(t)} \right). \quad (48)$$

This result means that in the considered case for $t > 0$ the following Theorem holds:

$$h_{11}(t) - h_{22}(t) = 0 \Leftrightarrow \frac{A_{12}(t)}{A_{21}(t)} = \text{const.}, \quad (t > 0). \quad (49)$$

Thus for $t > 0$ the problem under study is reduced to the Khalfin's Theorem (see the relation (39)).

From (45) and (46) it is easy to see that at $t = 0$

$$h_{jj}(0) = \langle \mathbf{j} | H | \mathbf{j} \rangle, \quad (j = 1, 2), \quad (50)$$

which means that in a CPT invariant system (27) in the case of pairs of unstable particles, for which transformations of type (28) hold

$$M_{11}(0) = M_{22}(0) \equiv \langle \mathbf{1} | H | \mathbf{1} \rangle, \quad (51)$$

the unstable particles "1" and "2" are created at $t = t_0 \equiv 0$ as particles with equal masses.

Now let us go on to analyze the conclusions following from the Khalfin's Theorem. CP noninvariance requires that $|r| \neq 1$ [7, 12, 13, 15] (see also [2] — [4], [10]). This means that in such a case there must be $r \equiv r(t) \neq \text{const.}$. So, if in the system considered the property (27) holds but

$$[\mathcal{CP}, H] \neq 0, \quad (52)$$

and the unstable states "1" and "2" are connected by a relation of type (28), then at $t > 0$ it must be $(h_{11}(t) - h_{22}(t)) \neq 0$ in this system. Assuming the LOY interpretation of $\Re(h_{jj}(t))$, ($j = 1, 2$), one can conclude from the Khalfin's Theorem and from the property (49) that if $A_{12}(t), A_{21}(t) \neq 0$ for $t > 0$ and if the total system considered is CPT-invariant, but CP-noninvariant, then $M_{11}(t) \neq M_{22}(t)$ for $t > 0$, that is, that contrary to the case of stable particles (the bound states), the masses of the simultaneously created unstable particle "1" and its antiparticle "2", which are connected by the relation (28), need not be equal for $t > t_0 = 0$. Of course, such a conclusion contradicts the standard LOY result (29), (30). However, one should remember that the LOY description of neutral K mesons and similar complexes is only an approximate one, and that the LOY approximation is not perfect. On the other hand the relation (49) and the Khalfin's Theorem follow from the basic principles of the quantum theory and are rigorous. Consequently, their implications should also be considered rigorous.

4 Beyond the LOY approximation

The approximate formulae for $H_{\parallel}(t)$ have been derived in [22, 23] using the Krolikowski–Rzewuski equation for the projection of a state vector [24], which

results from the Schrödinger equation (9) for the total system under consideration, and, in the case of the initial conditions of the type (10), takes the following form

$$(i\frac{\partial}{\partial t} - PHP)U_{\parallel}(t)|\psi >_{\parallel} = -i \int_0^{\infty} K(t - \tau)U_{\parallel}(\tau)|\psi >_{\parallel} d\tau, \quad (53)$$

where $U_{\parallel}(0) = P$,

$$K(t) = \Theta(t)PHQ \exp(-itQHQ)QHP, \quad (54)$$

and $\Theta(t) = \{1 \text{ for } t \geq 0, \ 0 \text{ for } t < 0\}$.

The integro-differential equation (53) can be replaced by the following differential one (see [17] — [24])

$$(i\frac{\partial}{\partial t} - PHP - V_{\parallel}(t))U_{\parallel}(t)|\psi >_{\parallel} = 0, \quad (55)$$

where

$$PHP + V_{\parallel}(t) \stackrel{\text{def}}{=} H_{\parallel}(t). \quad (56)$$

Taking into account (53) and (55) or (1) one finds from (4) and (53)

$$V_{\parallel}(t)U_{\parallel}(t) = -i \int_0^{\infty} K(t - \tau)U_{\parallel}(\tau)d\tau \stackrel{\text{def}}{=} -iK * U_{\parallel}(t). \quad (57)$$

(Here the asterisk, $*$, denotes the convolution: $f * g(t) = \int_0^{\infty} f(t - \tau)g(\tau) d\tau$). Next, using this relation and a retarded Green's operator $G(t)$ for the equation (53)

$$G(t) = -i\Theta(t) \exp(-itPHP)P, \quad (58)$$

one obtains [22, 23]

$$U_{\parallel}(t) = \left[1 + \sum_{n=1}^{\infty} (-i)^n L * \dots * L \right] * U_{\parallel}^{(0)}(t), \quad (59)$$

where L is convoluted n times, $1 \equiv 1(t) \equiv \delta(t)$,

$$L(t) = G * K(t), \quad (60)$$

$$U_{\parallel}^{(0)} = \exp(-itPHP) P \quad (61)$$

is a "free" solution of Eq. (53). Thus from (57)

$$V_{\parallel}(t) U_{\parallel}(t) = -iK * \left[1 + \sum_{n=1}^{\infty} (-i)^n L * \dots * L \right] * U_{\parallel}^{(0)}(t), \quad (62)$$

Of course, the series (59), (62) are convergent if $\| L(t) \| < 1$. If for every $t \geq 0$

$$\| L(t) \| \ll 1, \quad (63)$$

then, to the lowest order of $L(t)$, one finds from (62) [22, 23]

$$V_{\parallel}(t) \cong V_{\parallel}^{(1)}(t) \stackrel{\text{def}}{=} -i \int_0^{\infty} K(t - \tau) \exp[i(t - \tau)PHP] d\tau. \quad (64)$$

Thus [19, 21, 22, 23]

$$H_{\parallel}(0) \equiv PHP, \quad V_{\parallel}(0) = 0, \quad V_{\parallel}(t \rightarrow 0) \simeq -itPHQHP. \quad (65)$$

If the projector P is defined as in (13) and H has the following property

$$PHP \equiv m_0 P, \quad (66)$$

that is for

$$H_{12} = H_{21} = 0, \quad (67)$$

the approximate formula (64) for $V_{\parallel}(t)$ leads to the following form of Pe^{itPHP} ,

$$Pe^{itPHP} = Pe^{itm_0}, \quad (68)$$

and thus to

$$V_{\parallel}^{(1)}(t) = -PHQ \frac{e^{-it(QHQ - m_0)} - 1}{QHQ - m_0} QHP, \quad (69)$$

which leads to $V_{\parallel} \stackrel{\text{def}}{=} \lim_{t \rightarrow \infty} V_{\parallel}^{(1)}(t)$,

$$V_{\parallel} = -\Sigma(m_0). \quad (70)$$

This means that in the case (66)

$$H_{\parallel} = m_0 P - \Sigma(m_0), \quad (71)$$

and $H_{\parallel} = H_{LOY}$.

On the other hand, in the case

$$H_{12} = H_{21}^* \neq 0, \quad (72)$$

the form of Pe^{itPHP} is more complicated. For example in the case of conserved CPT, formula (64) leads to the following form of $V_{\parallel} \stackrel{\text{def}}{=} \lim_{t \rightarrow \infty} V_{\parallel}^{(1)}(t)$ [9, 25]

$$\begin{aligned} V_{\parallel}^{\Theta} = & -\frac{1}{2}\Sigma(H_0 + |H_{12}|) \left[\left(1 - \frac{H_0}{|H_{12}|}\right)P + \frac{1}{|H_{12}|}PH P \right] \\ & -\frac{1}{2}\Sigma(H_0 - |H_{12}|) \left[\left(1 + \frac{H_0}{|H_{12}|}\right)P - \frac{1}{|H_{12}|}PH P \right], \end{aligned} \quad (73)$$

where

$$H_0 \stackrel{\text{def}}{=} \frac{1}{2}(H_{11} + H_{22}), \quad (74)$$

and V_{\parallel}^{Θ} denotes V_{\parallel} when (27) occurs.

In the general case (72), when there are no assumptions on symmetries of the type CP-, T-, or CPT-symmetry for the total Hamiltonian H of the system considered, the form of $V_{\parallel} = V_{\parallel}(t \rightarrow \infty) \cong V_{\parallel}^{(1)}(\infty)$ is even more complicated. In such a case one finds the following expressions for the matrix elements $v_{jk}(t \rightarrow \infty) \stackrel{\text{def}}{=} v_{jk}$ of V_{\parallel} [22, 23],

$$\begin{aligned} v_{j1} = & -\frac{1}{2}\left(1 + \frac{H_z}{\kappa}\right)\Sigma_{j1}(H_0 + \kappa) - \frac{1}{2}\left(1 - \frac{H_z}{\kappa}\right)\Sigma_{j1}(H_0 - \kappa) \\ & - \frac{H_{21}}{2\kappa}\Sigma_{j2}(H_0 + \kappa) + \frac{H_{21}}{2\kappa}\Sigma_{j2}(H_0 - \kappa), \\ v_{j2} = & -\frac{1}{2}\left(1 - \frac{H_z}{\kappa}\right)\Sigma_{j2}(H_0 + \kappa) - \frac{1}{2}\left(1 + \frac{H_z}{\kappa}\right)\Sigma_{j2}(H_0 - \kappa) \\ & - \frac{H_{12}}{2\kappa}\Sigma_{j1}(H_0 + \kappa) + \frac{H_{12}}{2\kappa}\Sigma_{j1}(H_0 - \kappa), \end{aligned} \quad (75)$$

where $j, k = 1, 2$,

$$H_z = \frac{1}{2}(H_{11} - H_{22}), \quad (76)$$

and

$$\kappa = (|H_{12}|^2 + H_z^2)^{1/2}. \quad (77)$$

Hence, by (56)

$$h_{jk} = H_{jk} + v_{jk}. \quad (78)$$

It should be emphasized that all components of the expressions (75) are of the same order with respect to $\Sigma(\varepsilon)$.

In the case of preserved CPT-symmetry (27), one finds $H_{11} = H_{22}$ which implies that $\kappa \equiv |H_{12}|$, $H_z \equiv 0$ and $H_0 \equiv H_{11} \equiv H_{22}$, and [22, 23]

$$\Sigma_{11}(\varepsilon = \varepsilon^*) \equiv \Sigma_{22}(\varepsilon = \varepsilon^*) \stackrel{\text{def}}{=} \Sigma_0(\varepsilon = \varepsilon^*). \quad (79)$$

Therefore matrix elements v_{jk}^Θ of operator V_\parallel^Θ take the following form

$$\begin{aligned} v_{j1}^\Theta = & - \frac{1}{2} \left\{ \Sigma_{j1}(H_0 + |H_{12}|) + \Sigma_{j1}(H_0 - |H_{12}|) \right. \\ & \left. + \frac{H_{21}}{|H_{12}|} \Sigma_{j2}(H_0 + |H_{12}|) - \frac{H_{21}}{|H_{12}|} \Sigma_{j2}(H_0 - |H_{12}|) \right\}, \end{aligned} \quad (80)$$

$$\begin{aligned} v_{j2}^\Theta = & - \frac{1}{2} \left\{ \Sigma_{j2}(H_0 + |H_{12}|) + \Sigma_{j2}(H_0 - |H_{12}|) \right. \\ & \left. + \frac{H_{12}}{|H_{12}|} \Sigma_{j1}(H_0 + |H_{12}|) - \frac{H_{12}}{|H_{12}|} \Sigma_{j1}(H_0 - |H_{12}|) \right\}, \end{aligned}$$

Assuming

$$|H_{12}| \ll |H_0|, \quad (81)$$

we find

$$v_{j1}^\Theta \simeq -\Sigma_{j1}(H_0) - H_{21} \frac{\partial \Sigma_{j2}(x)}{\partial x} \bigg|_{x=H_0}, \quad (82)$$

$$v_{j2}^\Theta \simeq -\Sigma_{j2}(H_0) - H_{12} \frac{\partial \Sigma_{j1}(x)}{\partial x} \bigg|_{x=H_0}, \quad (83)$$

where $j = 1, 2$. One should stress that due to the presence of resonance terms, derivatives $\frac{\partial}{\partial x} \Sigma_{jk}(x)$ need not be small and the same is true about products $H_{jk} \frac{\partial}{\partial x} \Sigma_{jk}(x)$ in (82), (83). Finally, assuming that (81) holds and using relations (82), (83), (78) and the expression (24), we obtain for the CPT-invariant system [26, 27]

$$h_{j1}^\Theta \simeq h_{j1}^{\text{LOY}} - H_{21} \frac{\partial \Sigma_{j2}(x)}{\partial x} \bigg|_{x=H_0} \stackrel{\text{def}}{=} h_{j1}^{\text{LOY}} + \delta h_{j1}, \quad (84)$$

$$h_{j2}^\Theta \simeq h_{j2}^{\text{LOY}} - H_{12} \frac{\partial \Sigma_{j1}(x)}{\partial x} \bigg|_{x=H_0} \stackrel{\text{def}}{=} h_{j2}^{\text{LOY}} + \delta h_{j2}, \quad (85)$$

where $j = 1, 2$. From these formulae we conclude that, e.g., the difference between the diagonal matrix elements of H_{\parallel}^{Θ} which plays an important role in designing CPT-invariance tests for the neutral kaons system, equals

$$\Delta h \stackrel{\text{def}}{=} h_{11} - h_{22} \simeq H_{12} \frac{\partial \Sigma_{21}(x)}{\partial x} \Big|_{x=H_0} - H_{21} \frac{\partial \Sigma_{12}(x)}{\partial x} \Big|_{x=H_0} \neq 0. \quad (86)$$

5 Final remarks

In the case of conserved CPT- and violated CP- symmetries there must be

$$h_{11}^{\Theta}(t) - h_{22}^{\Theta}(t) \neq 0 \text{ for } t > t_0 = 0,$$

and, $h_{11}(0) = h_{22}(0) = \langle \mathbf{1} | H | \mathbf{1} \rangle$, for the exact H_{\parallel} .

Note that properties of the more accurate approximation described in Sec. 4 are consistent with the general properties and conclusions obtained in Sec. 3 for the exact effective Hamiltonian — compare (65) and (50) and relations (49) with (86).

From the result (86) it follows that $\Delta h = 0$ can be achieved only if $H_{12} = H_{21} = 0$. This means that if the first order $|\Delta S| = 2$ interactions are forbidden in the K_0, \overline{K}_0 complex then predictions following from the use of the mentioned more accurate approximation and from the LOY theory should lead to the the same masses for K_0 and for \overline{K}_0 . This does not contradict the results of Sec. 3 derived for the exact H_{\parallel} : the mass difference is very, very small and should arise at higher orders of the more accurate approximation.

On the other hand from (86) it follows that $\Delta h \neq 0$ if and only if $H_{12} \neq 0$. This means that if measurable deviations from the LOY predictions concerning the masses of, e.g. K_0, \overline{K}_0 mesons are ever detected, then the most plausible interpretation of this result will be the existence of first order $|\Delta S| = 2$ interactions in the system considered.

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